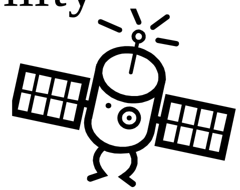
Satellite Meteorology -3 (May 10)
Satellite Orbits

Lecturer: Hiro Masunaga (ISEE)

Satellite orbits (from the previous class)

- Orbit configurations for Earth observing satellites
  - Geostationary/Geosynchronous orbit（静止軌道）
    - Orbiting at the altitude of ~36,000 km above the equator
    - Continuously observes (a certain side of) the Earth
    - Drawbacks: costly, and limitations in sensor capability



  - Low-Earth orbiting (LEO) orbit （低軌道）
    - Flying at a range of altitudes of a few 100-1000 km
    - Mainly polar orbiting (sun-synchronous) satellites
      - Always observes at a fixed hour in local time.
    - Some are sun-asynchronous (TRMM and GPM)
    - Drawbacks: overpasses only 2-3 times per day at best.

Newton's law of gravity

- The law of universal gravitation
  - *a*: gravitational acceleration (重力加速度), *m*: mass (質量)
    r: distance from the Earth center (地球中心からの距離)

$$ma = \frac{GM_e m}{r^2}$$

$G = 6.67 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$:gravitation constant (万有引力定数)

$M_e = 5.97 \times 10^{24}$ kg : the mass of the Earth (地球の質量)

- Q. Estimate the gravitational acceleration at the Earth surface, g.
  - Note) R_e=6371 km: the Earth radius (地球半径)

Determination of satellite orbital radius

- Orbital radius of satellites: *r*
  - ω: orbital angular velocity of satellite (衛星の公転角速度)

$$mr\omega^2 = \frac{GM_e m}{r^2}$$

- Q. Estimate the orbital altitude (軌道高度) of geostationary satellites.
  - Note -1) Orbital period, T, is given by $T = \frac{2\pi}{\omega}$
  - Note -2) Orbital altitude = Distance between the satellite and the Earth *surface*.

Orbital period of LEO satellites

- The "Centrifugal force=Gravity" relation gives the Kelper's third law
  - Orbital period $T = \frac{2\pi}{\omega} = 2\pi\left[\frac{a^3}{GM_e}\right]^{1/2}$
- Typical orbital periods of LEO satellites
  - Orbital altitude = 400km → Orbital period = 92 min.
  - Orbital altitude = 800km → Orbital period = 101 min.
  - LEO satellites fly around the Earth about 14-16 times per day (e.g., 90 min.× 16 = 24 hours).

Perturbations to the two-body gravitation

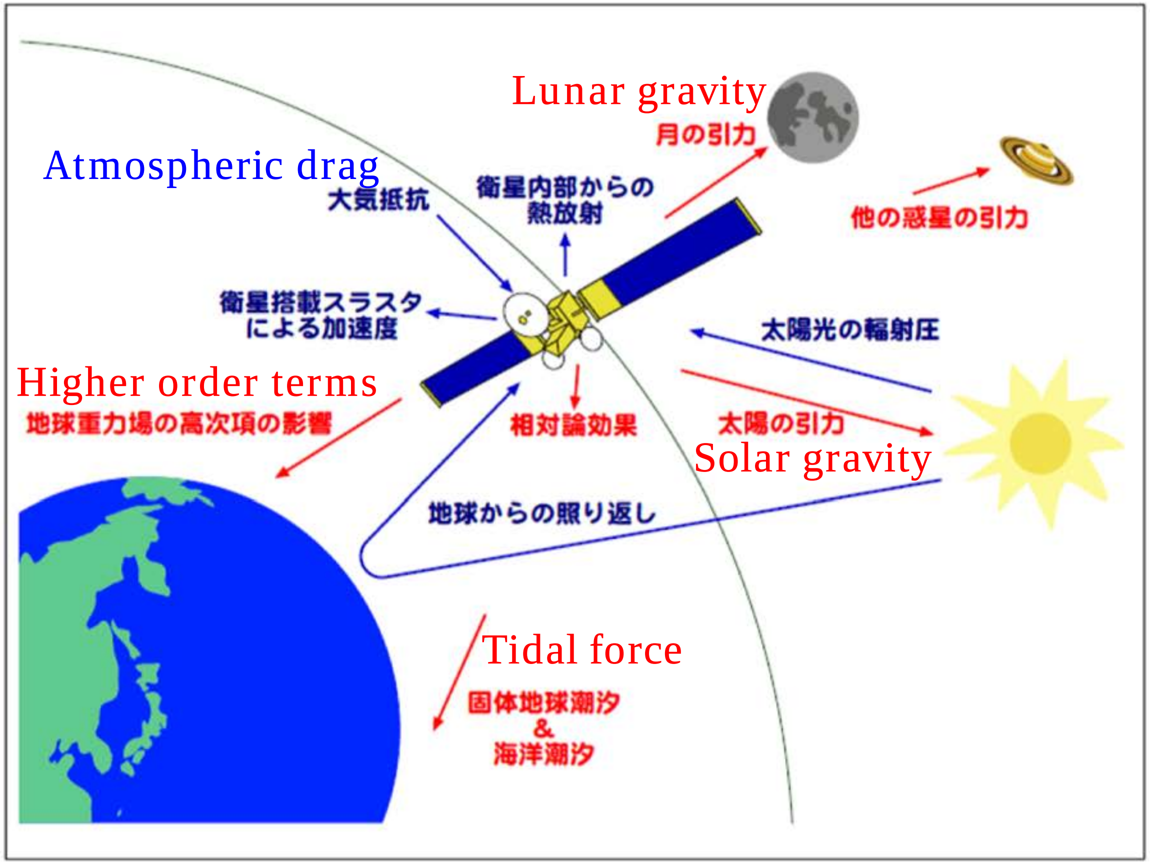


Fig. 1 人工衛星に作用する様々な摂動力。青字で書かれた項目は、衛星の形状や姿勢等に依存するもの。

図：東大地震研　http://www.eri.u-tokyo.ac.jp/KOHO/HIGHLIGHT/KYODO/2004-W-01/ppr/eri0411-04kubo-oka.pdf

Relative importance of the perturbations

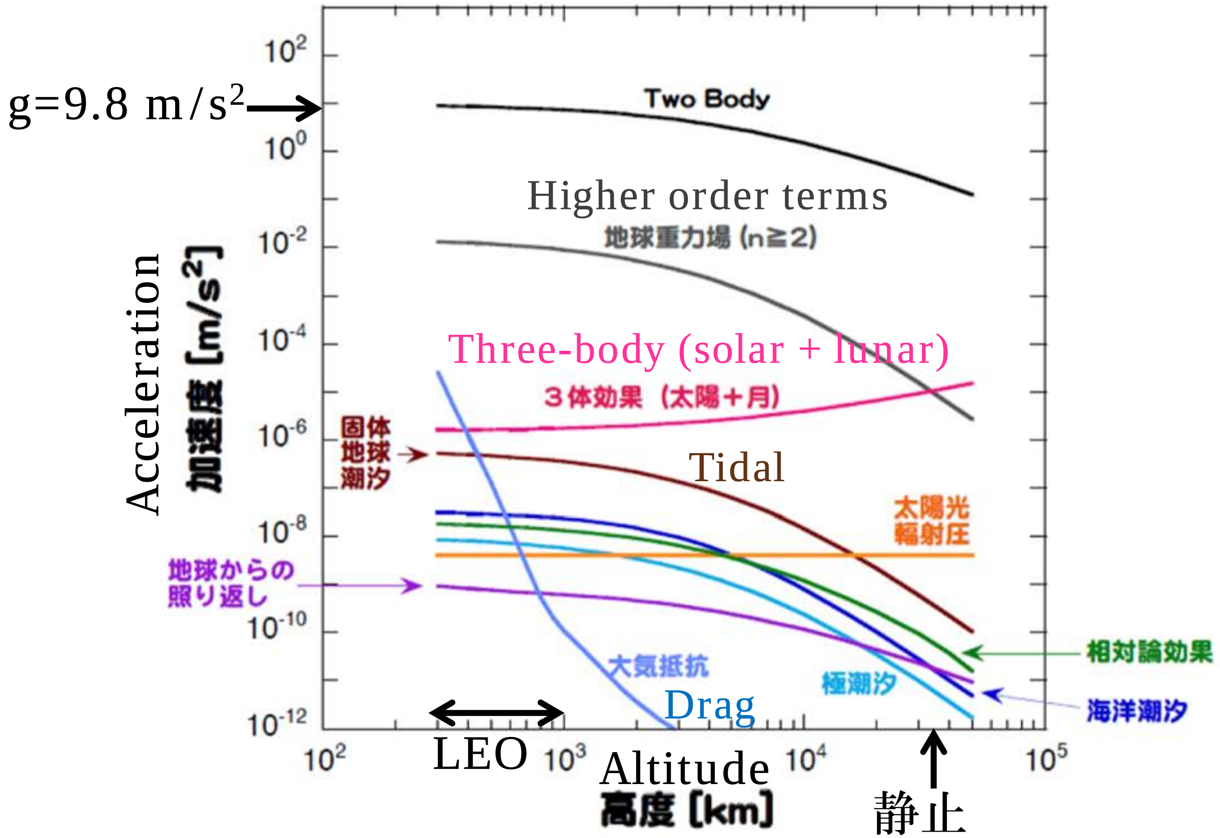


Fig. 2 人工衛星に作用する摂動と高度の関係。Fig. 1 で青い矢印で示した衛星の形状に依存する摂動に関しては、(断面積) / (質量) 比を 0.0093 と仮定して計算した。

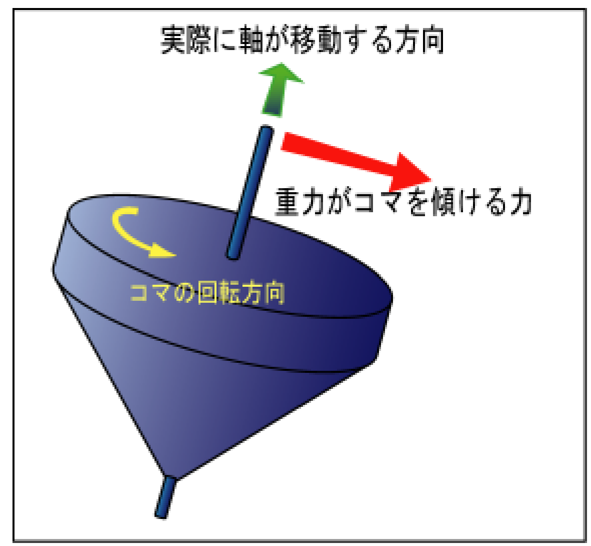
図：東大地震研　http://www.eri.u-tokyo.ac.jp/KOHO/HIGHLIGHT/KYODO/2004-W-01/ppr/eri0411-04kubo-oka.pdf

Kepler's 1st law and the two-body problem

- Kepler's 1st law
  - The orbit of a planet is an ellipse with the Sun at one of the two foci. (惑星軌道は太陽を焦点の一つとする楕円)
  - This would be exactly true if the Earth were a perfect sphere. In reality, the Earth is an ellipsoid (楕円体).

Rotation of the orbital plane

- Gyroscopic effect
  - The satellite is not pulled exactly toward the Earth center.
    - The orbit is forced to be tilted toward the equatorial plane.
  - The gyro effect makes the orbital plane slowly rotate.



http://homepage2.nifty.com/eman/dynamics/topspin.html

Non-sphericity of the gravitational field

- Representation of spherical asymmetry
  - The Legendre expansion of the gravitational field, *U*.

$$U = -\frac{GM_e}{r}\sum_{n=0}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\sin\varphi)$$

  - Expansion coefficients
    - n=0: Spherical symmetry component, $J_0=1$.
    - n=1: Offset of the center of gravity. Can be always set to be $J_1=0$.
    - n=2: Ellipticity. For the Earth, $J_2=0.00108263$
    - n>2: Even higher order terms $J_n \ll J_2$
  - Legendre polynomials $P_0(x)=1, \quad P_1(x)=x, \quad P_2(x)=\frac{1}{2}(3x^2-1), \cdots$

The precession of the orbital plane by J_2

- Orbital precession (歳差) rate due to the J_2 term.

$$\dot{\Omega} = -\frac{3}{2} J_2 \left[\frac{R_e}{a(1-e^2)} \right]^2 n \cos i$$

i: orbital inclination (軌道傾斜角)

n: mean motion (平均運動) or mean angular velocity

$$n \approx \left[\frac{GM_e}{a^3} \right]^{1/2}$$

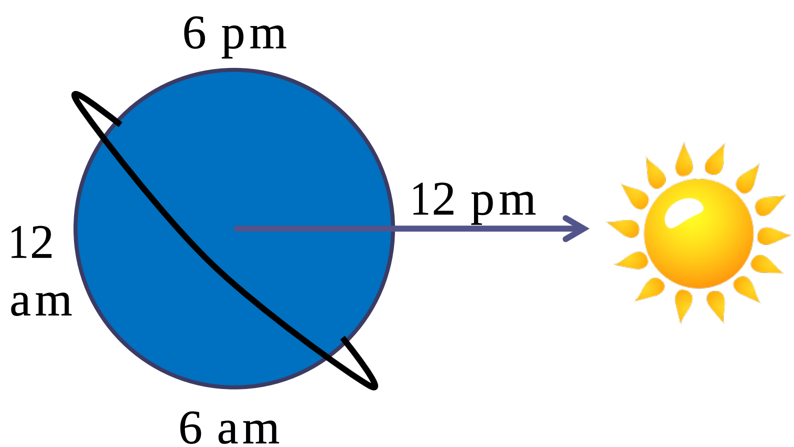
（Kepler's 3rd law）

$$\Rightarrow \quad \dot{\Omega} = -\frac{1.32 \times 10^{18}}{a^{7/2} (1-e^2)^2} \cos i \ \ [\mathrm{rad/s}]$$

a: orbital radius [m]

Sun-synchronous (太陽同期) satellites

- Sun-synchronous orbit
  - The orbit whose plane has a fixed angle against the sun.
  - Overpasses at **a same local time** twice a day (am/pm)



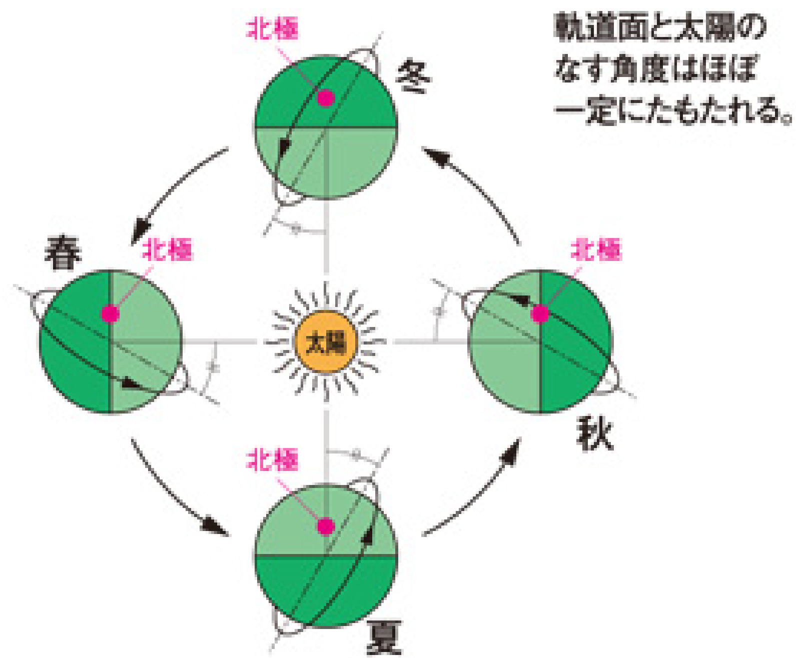


図　http://www.eorc.jaxa.jp/hatoyama/experience/rm_kiso/satellit_type_orbit.html

Sun-synchronous (太陽同期) satellites

- Sun-synchronous orbit
  - The orbit whose plane has a fixed angle against the sun.

$$\dot{\Omega} = \frac{2\pi}{Y_\theta}, \quad Y_\theta = 365.25\cdots \text{days}$$

$$i \approx 98^\circ \quad \text{for } a = 7000 \text{ km}, \ e = 0$$

(orbital altitude ~630 km)

  - “Polar-orbiting” satellite

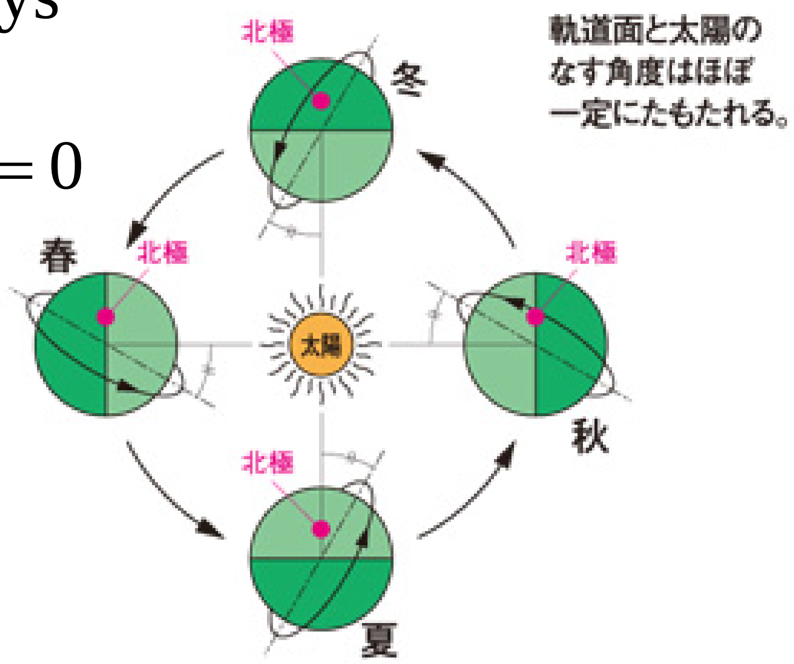
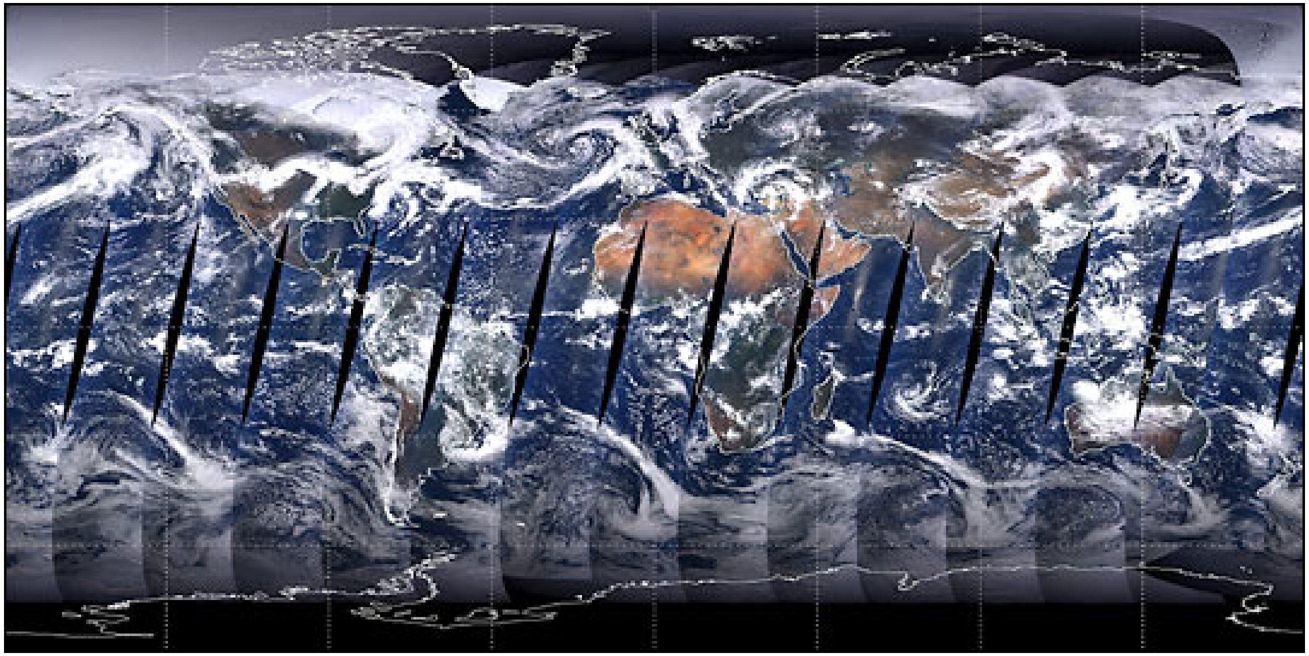


図　http://www.eorc.jaxa.jp/hatoyama/experience/rm_kiso/satellit_type_orbit.html

Observations by a sun-synchronous satellite

- MODIS visible images
  - Terra MODIS 10:30 am overpasses (for one day)
  - Same local time→ Same clouds have the same brightness.



Sun-asynchronous (太陽非同期) satellites

- Drawbacks of sun-synchronous satellites
  - Diurnal cycle is not observable.
  - Overpasses are relatively infrequent over low latitudes.
- Sun-asynchronous satellites
  - TRMM (熱帯降雨観測衛星)
    - Japan/US mission. Orbital inclination i=35°
  - GPM (全球降水観測計画)
    - Follow-on to TRMM. Orbital incliniation i=65°
  - Megha-Tropiques
    - French/Indian mission. Orbital inclination i=20°
  - Most of the other Earth observing satellites for atmospheric and oceanic sciences are in sun-synchronous orbit.

Summary

- Physical factors responsible for satellite orbits
  - The central principle: a two-body problem (Kepler's laws)
  - Perturbations: non-sphericity etc.
- Geosynchronous orbit
  - Two-body problem with the orbital period of 1 day.
- Low-Earth orbits
  - Sun-synchronous orbits (~ polar orbits)
    - Precession is designed to have the period of ~365.25 days.
  - Sun-asynchronous orbits
    - Targeted on diurnal variability and low-latitude observations.